Study of the nuclear equation of state from the rapidity-dependent elliptic flow

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Outline

- **Introduction**
  Why do we need to study the nuclear EoS?
  The status of determination of the incompressibility $K_0$.
  Observable: elliptic flow.

- **Transport model**
  Ultrarelativistic Quantum Molecular Dynamics (UrQMD) Model

- **Results**

- **Summary**
Nuclear equation of state

\[ E(\rho, \delta) = E(\rho, 0) + E_{\text{sym}}(\rho) \delta^2 + O(\delta^4), \]

\[ E(\rho, 0) = E_0 + \frac{K_0}{2} \left( \frac{\rho - \rho_0}{3 \rho_0} \right)^2 + \ldots, \]

\[ E_{\text{sym}}(\rho) = S_0 + L \left( \frac{\rho - \rho_0}{3 \rho_0} \right) + \frac{K_{\text{sym}}}{2} \left( \frac{\rho - \rho_0}{3 \rho_0} \right)^2 + \ldots \]

\[ K_0 = 9 \rho_0^2 \left( \frac{\partial^2 E}{\partial \rho^2} \right) \bigg|_{\rho = \rho_0} \]

\[ L = 3 \rho \left( \frac{dE_{\text{sym}}(\rho)}{d \rho} \right) \bigg|_{\rho = \rho_0} \]

\( K_0 \) and \( L \) determine the EOS in the vicinity of the saturation density. 3
The incompressibility $K_0$ from properties of nuclei

Analyses from properties of nuclei
37 constraints collected in PRC89.044316

1. GMR energies
2. Nuclear masses
3. Charge radius

$250 < K_0 < 315$ MeV, based on the most precise and up-to-date data on GMR energies.

The incompressibility $K_0$ from Heavy-ion collision

1. Kaon production
2. Collective flow

Zhao-Qing Feng, PRC83.067604 (2011)
……
The incompressibility $K_0$ from Heavy-ion collision

1. Kaon production
2. Collective flow

Rule out the most extreme $K_0$ (less than 167 MeV or larger than 380 MeV) for EOS.

Observable: elliptic flow

\[ v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle \]

\[ v_2 = \begin{cases} 
-1 & \text{Full out} \\
0 & \text{Spherical} \\
+1 & \text{Full in}
\end{cases} \]

Coordinate-space anisotropy

Transverse Plane

Momentum -space anisotropy

In-plane
\[ v_2 > 0 \]

Out-of-plane
\[ v_2 < 0 \]

\[ p_t = \sqrt{p_x^2 + p_y^2}; \quad y = \frac{1}{2} \ln \left[ \frac{E + p_z}{E - p_z} \right] \]
Observable: elliptic flow at mid-rapidity ($y_z=0$)
Elliptic flow: transverse momentum and rapidity dependence

System: Ca+Ca, Ni+Ni, Ru+Ru, Zr+Zr, Xe+Sn, Au+Au
Beam energies: 90-1500 MeV/nucleon
Multi dependences: rapidity, transverse momentum, centrality, particle species

A large amount of elliptic flow data in HICs at intermediate energies has been presented by FOPI Collaboration in the recent decade.

What can we learn from those new precise and plentiful elliptic flow data?

The UrQMD model is a microscopic model used to simulate (ultra)relativistic HICs in the energy range from Bevalac and SIS up to AGS, SPS and RHIC, LHC.

**Lorentz-covariant dynamics**

1. covariant dynamics
2. improved and extended collision term

**UrQMD**

- Collision term
- Mean field term

**RQMD**

**QMD**

**improve and extend collision term**

JPG 32.407-415, PRC 83.044617
1). Initialization
   Get the coordinate \( r \) and the momentum \( p \)

2). Propagation
   Nucleon moves in the mean-field.
   Density, momentum, isospin-dependent.

3). Collision term
   Medium modified cross section.
   Density, momentum, isospin-dependent. Pauli blocking.

4). Cluster recognition
   an isospin-dependent Minimum Spanning Tree.
   With an appropriate choice of the in-medium nucleon–nucleon cross section, the recent published experimental data can be reproduced fairly well.

\[ \phi_i(\vec{r}_i; t) = \frac{1}{(2\pi)^{3/2}(\Delta x)^{3/2}} \exp \left\{ -\frac{(\vec{r}_i - \vec{R}_i(t))^2}{(2\Delta x)^2} + i\vec{r}_i \cdot \vec{P}_i(t) \right\} \]

\[ \dot{p}_i = -\frac{\partial H}{\partial r_i}, \quad \dot{r}_i = \frac{\partial H}{\partial p_i}. \]

Input: \( f\sigma_{pp} = f\sigma_{nn}, \) and \( f\sigma_{np} \)

Input Skyrme forces
Symmetry energy

PRC 83, 044617; 89.034606; 89.044603
The rapidity-dependent directed and elliptic flow

PRC 83, 044617 ; 89.034606;89.044603
The rapidity-dependent directed and elliptic flow

PRC 83, 044617 ; 89.034606;89.044603
The rapidity-dependent elliptic flow

**Table 1**
Saturation properties of nuclear matter as obtained with selected Skyrme parametrizations used in this work.

<table>
<thead>
<tr>
<th></th>
<th>$K_0$ (MeV)</th>
<th>$S_0$ (MeV)</th>
<th>$L$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skxs15</td>
<td>201</td>
<td>31.88</td>
<td>34.79</td>
</tr>
<tr>
<td>MSK1</td>
<td>234</td>
<td>30.00</td>
<td>33.92</td>
</tr>
<tr>
<td>SKX</td>
<td>271</td>
<td>31.10</td>
<td>33.18</td>
</tr>
<tr>
<td>SV-sym34</td>
<td>234</td>
<td>34.00</td>
<td>80.95</td>
</tr>
</tbody>
</table>

**Figure**

- $E_{lab}=0.4A$ GeV
- $E_{lab}=0.6A$ GeV
- $E_{lab}=0.8A$ GeV
- $E_{lab}=1.0A$ GeV

For soft, $v_2 = v_{20} + v_{22}y_0^2$; for stiff, $v_{2n} = |v_{20}| + v_{22}$.
Results: $v_{2n}$ of protons

1. $v_{2n}$ increases strongly with increasing $K_0$ in both the IQMD and UrQMD model calculations, though this slope dependence is not exactly the same for two models.

2. The results of the UrQMD model exhibit an approximate linearity between the $v_{2n}$ and $K_0$. Central value of $K_0$ is $240 \pm 20$ and $275 \pm 25$ MeV for FP4 and FP5, respectively.

3. The central value of obtained $K_0$ is about 220 MeV in the IQMD. The difference comes from the collision term in the two models, i.e., the free NN cross section is used in IQMD, and also different Pauli blocking.

4. The results for $v_{2n}$ calculated with SV-sym34 and MSK1, with the same value of $K_0$, but different slope parameter $L$, are very close to each other, it illustrates the $v_{2n}$ is much more sensitive to $K_0$ than $L$. 

\[ E_{\text{lab}} = 0.4 \text{A GeV} \quad E_{\text{lab}} = 0.6 \text{A GeV} \]

\[ E_{\text{lab}} = 0.8 \text{A GeV} \quad E_{\text{lab}} = 1.0 \text{A GeV} \]
Results: $v_{2n}$ of deuterons

By averaging over the four energies, we derive $K_0=190 \pm 10$ MeV ($K_0 = 225 \pm 20$ MeV) for the FU3FP4 (FU3FP5) parametrization, results that are larger than the $K_0=170 \pm 8$ MeV deduced with the IQMD model.

Overall, the analysis of the rapidity dependence of the elliptic flow supports the soft choice for the nuclear equation of state.

With the FU3FP4 parametrization (i.e., the preferred choice in the present version of the UrQMD model) of the in-medium NN cross section, $K_0 = 240 \pm 20$ MeV and $K_0=190 \pm 10$ MeV are extracted from the $v_{2n}$ of free protons and deuterons, respectively. By combining the error intervals of the proton and deuteron results, an averaged $K_0 = 220 \pm 40$ MeV is obtained.
Summary

- By comparing the UrQMD model calculations with the recent FOPI data for the elliptic flow in Au + Au collisions, it is found that the nuclear incompressibility $K_0$ is quite sensitive to the $v_{2n}$, a quantity obtained from a quadratic fit ($v_2 = v_{20} + v_{22} \cdot y_0^2$) of the elliptic flow as a function of rapidity by adding the coefficients as $v_{2n} = |v_{20}| + |v_{22}|$.

- Using the elliptic flow $v_{2n}$ of protons and deuterons, the incompressibility of symmetric matter $K_0 = 220 \pm 40$ MeV can be obtained. Overall, both UrQMD and IQMD models support a soft choice of nuclear equation of state.

Thank you very much for your attention!
The in-medium NN cross section

FU3FP4

FU3FP5

\( p/\rho_0 \)

\( p \ (\text{GeV}/c) \)

\( p \ (\text{GeV}/c) \)
Observable: elliptic flow

Flow refers to how energy, momentum, and number of particles varies with direction.

\[
v_2 = \sqrt{\frac{p_x^2 - p_y^2}{p_t^2}}
\]

\[
\begin{align*}
  v_2 & = -1 \quad \text{full out} \\
  v_2 & = 0 \quad \text{spherical} \\
  v_2 & = +1 \quad \text{full in}
\end{align*}
\]

In noncentral heavy-ion collisions

\[
p_t = \sqrt{p_x^2 + p_y^2}
\]

\[
y = \frac{1}{2} \ln \left[ \frac{E + p_z}{E - p_z} \right]
\]

negative \( v_2 \), a preferential out-plane emission

positive \( v_2 \), a preferential in-plane emission
out of plane emission

semi central collisions

out of plane emission

bounce off
Observable: elliptic flow at mid-rapidity ($y_z=0$)

In-plane elliptic flow

bounce-off

squeeze-out

Elliptic Flow

$v_2$ vs $E_{beam}/A$ (GeV)
The incompressibility $K_0$ from Heavy-ion collision

A new observable $v_{2n}$ which characterizes the $v_2$ in a wide rapidity range was proposed, and found that $v_{2n}$ is very sensitive to EOS.

$$v_{2n} = |v_{20}| + v_{22}$$