Investigation of Giant Dipole Resonances in Heavy Deformed Nuclei with EQMD Model

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Outline

◆ **Background**

◆ **Model and Method**

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◆ **Summary**
I. Background—Giant Dipole Resonance (GDR)

- In 1947, Giant Dipole Resonance (GDR) was firstly discovered by Baldwin and Klaiber.
- In 1948, Goldhaber and Teller explained GDR as dipole vibrations of collective motion of all protons vs all neutrons inside the nucleus, assuming periodic displacement of a rigid proton sphere relative to a rigid neutron sphere.
- In 1950, Steinwedel and Jensen treated GDR as a density motion of the proton fluid with density, and neutron fluid with density under the condition of constant total density.

Absorption of high-energy photons shows a resonance-like behavior.

Fig 1. X-ray fission yield for uranium

I. Background—Giant Dipole Resonance (GDR)

Microscopically, GDR is described as a coherent superposition of 1 particle-1 hole excitations resulting from the action of an electromagnetic operator on the nuclear ground state.

Fig 1. Schematic representation of E1 and E2 single particle transitions between shell model state

I. Background—Giant Dipole Resonance (GDR)

- GDR has been turned out to be a general feature of all nuclei.
- GDR has been successfully applied for studying different α cluster configurations.

Fig 1. GDR spectra with different cluster configuration

Fig 2. Coherence dipole motion of $^{16}$O kite structure.

I. Background—GDR in deformed nuclei

- Deformation of a nucleus that have an ellipsoidal shape is described in terms of quadrupole deformation parameter $\beta_2$.

$$\beta_2 = \sqrt{\frac{4\pi}{5} \frac{R_x - R_0}{R_0}}.$$

- GDR is split into two components in deformed nuclei.

- The splitting is attributed to different frequencies of dipole oscillations along the major and minor axes of a deformed nucleus.

- Many methods have been used to study GDR in deformed nuclei
  - Photoabsorption, $\gamma$-decay, inelastic scattering
  - Random phase approximation approach, Time dependent Skyrme-Hartree-Fock method, BUU, IQMD, EQMD

In EQMD, the total wave function of the system as a direct product of Gaussian wave packets of nucleons

\[ \Psi = \prod_i \phi_i(r_i), \]

\[ \varphi_i(r_i) = \left( \frac{v_i + v_i^*}{2\pi} \right)^{3/4} \exp \left[ -\frac{v_i}{2} (r_i - R_i)^2 + \frac{i}{\hbar} P_i \cdot r_i \right], \]

where \( v_i = 1/\lambda_i + i\delta_i \) is width of the complex Gaussian wave packets. \( \lambda \) and \( \delta \) are dynamic variables. The \( v_i \) of Gaussian wave packets for each nucleon is dynamic and independent.

The Hamiltonian can be written as

\[ H = \langle \Psi | \sum_i -\frac{\hbar^2}{2m} \nabla_i^2 - \hat{T}_{c.m.} + \hat{H}_{int} | \Psi \rangle \]

\[ = \sum_i \left[ \frac{P_i^2}{2m} + \frac{3\hbar^2}{4m\lambda_i} \left( 1 + \frac{\lambda_i^2 \delta_i^2}{4\lambda_i} \right) \right] - T_{c.m} + H_{int}. \]

\( H_{int} \): The potential energy term, including Pauli potential

\( T_{c.m} \): The spurious zero-point center-of-mass kinetic energy

II. Model—EQMD model

The energy-minimum states of initial ground nuclei are obtained by starting from a random configuration and by solving the damped equations of motion as

\[
\begin{align*}
\dot{R}_i &= \frac{\partial H}{\partial P_i} + \mu_R \frac{\partial H}{\partial R_i}, \\
\dot{P}_i &= -\frac{\partial H}{\partial R_i} + \mu_p \frac{\partial H}{\partial P_i}, \\
\frac{3\hbar}{4} \lambda_i &= -\frac{\partial H}{\partial \delta_i} + \mu_\lambda \frac{\partial H}{\partial \lambda_i}, \\
\frac{3\hbar}{4} \delta_i &= \frac{\partial H}{\partial \lambda_i} + \mu_\delta \frac{\partial H}{\partial \delta_i}.
\end{align*}
\]

Here \(\mu_R, \mu_p, \mu_\lambda,\) and \(\mu_\delta\) are damping coefficients.
All the nucleon wave packets stop their motions at the energy-minimum state,

\[
\begin{align*}
\dot{R}_i &= 0, & \dot{P}_i &= 0, \\
\dot{\lambda}_i &= 0, & \dot{\delta}_i &= 0,
\end{align*}
\]

while in the standard QMD model nucleons are moving around in the ground states.

II. Method —— Calculation method of GDR

The dipole moments of the system in coordinate space $D_G(t)$ and momentum space $K_G(t)$ are defined as follows

$$D_G(t) = \frac{NZ}{A} X(t) = \frac{NZ}{A} (R_Z - R_N)$$
$$K_G(t) = \frac{NZ}{A\hbar} \Pi(t) = \frac{NZ}{A\hbar} (P_Z - P_N)$$

From the Fourier transform of the second derivative of $D_G(t)$ with respect to time, i.e.,

$$D''(\omega) = \int_{t_0}^{t_{\text{max}}} D''_G(t)e^{i\omega t} dt.$$

The dipole resonance strength of the system at excitation energy $E = \hbar \omega$ can be obtained by

$$\frac{dP}{dE_\gamma} = \frac{2e^2}{3\pi\hbar c^3 E_\gamma} |D''(\omega)|^2,$$

II. Method —— GDR in deformed nuclei

- In this work, we calculate the GDRs along the x and z axes, respectively.
- In analogy to the superposition of two noninterfering Lorentz lines for statically deformed nuclei for fitting the experimental data, we take the method of the superposition of two GDR spectra to gain the total resonance strength in a deformed nucleus. The formula is given as

\[ \frac{dP}{dE_\gamma} = \sum_{i=1}^{2} \frac{\left( \frac{dP}{dE_\gamma} \right)_{mi}}{1 + \left( \frac{E_\gamma^2 - E_{mi}^2}{E_{mi}^2 \Gamma_{mi}^2} \right)^2} \]

The resonance strength maximum

The peak position of the resonance maximum

The resonance width at half-maximum

\( i = 1,2 \) correspond to the x- and z-axis resonance components of the deformed nucleus. Note that x-axis is the major axis and z-axis corresponds to minor axis.
III. Results and Discussion

Fig 1. The time evolution of the dipole moments for the chain of Sm isotopes

- All of the dipole oscillations are symmetrical around \( D_G(K_G) = 0 \).
- The resonance frequencies along the x-axis direction are lower than those in z-axis direction.

Fig 2. Dipole strengths in the Nd isotopes and Sm isotopes
III. Results and Discussion

\[ \frac{dP}{dE_\gamma} = \sum_{i=1}^{2} \frac{\left( \frac{dP}{dE_\gamma} \right)_{mi}}{1 + \frac{(E_{\gamma} - E_{mi})^2}{E_{mi}^2 \Gamma_{mi}^2}} \]

- With the decreasing of the deformation of nuclei, the GDR width also decrease.
- The calculated GDRs can perfectly reproduce the shape of the GDR spectra.
- The results confirm the reliability of the methods and the model to study the GDR in deformed nuclei.
- It predicts the shapes of GDR spectra in Sm (A = 134,136,138) and Nd (A = 130,132,134) isotopes, which is possible to be verified by experiments.

Fig. The total GDR spectra in the isotopic chain of Sm and Nd isotopes
### III. Results and Discussion

**Fig 1.** The mass dependence of peak position of GDRs along the $x$- and $z$-axis.

**Fig 2.** The correlation between $\Delta E_m/\bar{E}_m$ and deformation parameter $\beta_2$.

- The resonance energy ($E_m$) in deformed nuclei is sensitive to the deformed parameter.

**Graphical Data:**
- **Sm** and **Nd** are shown with different markers.
- The graphs illustrate the trend of $\Delta E_m/\bar{E}_m$ with $\beta_2$.
- Equations for the trend lines: $y = 0.69402x - 0.00869$ and $y = 0.53845x + 0.02872$.
III. Results and Discussion

Fig. The dependence of the GDR spectra on symmetry energy coefficient ($E_{\text{sym}}$) in heavy deformed nuclei $^{150}\text{Nd}$

- With the increasing of $E_{\text{sym}}$ from 30 to 34 MeV, the GDR spectra of the system have the obvious trend of moving to the right; i.e., The energy position of GDR is governed by the symmetry energy.
- The value of 32 MeV is the best choice for the symmetry energy coefficient to investigate the GDRs in heavy deformed nuclei in the framework of the EQMD model.
IV. Summary

Summary

The EQMD model has a light of success for describing the GDRs in heavy deformed nuclei.

- It can quite well reproduce the shape of GDR spectra from spherical to prolate shapes.
- The resonance energy \(E_m\) in deformed nuclei is sensitive to the deformed parameter.
- The splitting of the GDR spectra is proportional to the deformation.

The calculated GDR spectra in the EQMD model are perfectly consistent with the experimental results when \(E_{sym} = 32\) MeV.

EQMD model might also be applied to treat the pygmy dipole resonance (PDR).
Thank you for your attention!