Extracting condensation fraction from multi-pion HBT correlations of the sources with Bose-Einstein condensation

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Introduction

- HBT interferometry is a tool to study the space & time of particle-emitting sources.
- Because HBT correlation occurs only for chaotic sources. So, it can be used to probe the source chaotic degree.
- Pions are the most copiously produced particles in high-energy collisions. In the heavy-ion collisions at the RHIC and LHC, the detected identical pions are about hundreds and thousands.

\[ \langle N_{\text{part}} \rangle = 383, \quad dN_{\text{ch}} / d\eta = 1601, \quad N_{\text{ch}} \approx 1601 \times 4 = 6400 \]

The high pion event multiplicity may possibly lead to occurrence of the pion condensate in ultra-relativistic heavy-ion collisions.
Three-pion HBT measurements for the $\sqrt{s_{NN}} = 2.76$ TeV Pb-Pb at the LHC (PRC89, 2014, 024911) indicate that the pion sources are not chaotic completely and with the considerable degree of coherence.

$r_3 < 2$ Partially coherent sources

Clear suppression for low $K_{T,3}$

Much more consistent for high $K_{T,3}$
Recently, the three and four-pion HBT measurements for the $\sqrt{S_{NN}} = 2.76$ TeV Pb-Pb at the LHC (PRC93,054908,2016) also indicate that the pion sources are not chaotic completely and with considerable degree of coherence.

20-40 % coherent fraction from the recent multi-pion measurement at ALICE

Why ?

It is our motivation to study the possible pion Bose-Einstein condensation in ultra-relativistic heavy-ion collisions and investigate the effects of the condensation on pion HBT measurements.
We consider an expanding pion source, the time-dependent harmonic oscillator potential is given by

\[ V(r, t) = \frac{1}{2} m \omega^2(t) r^2 = \frac{1}{2} \hbar \omega(t) \frac{r^2}{a^2(t)}, \]

Assuming the system relaxation time is smaller than source evolution time, we may deal the pion gas as a quasi-static adiabatic expansion, and have

\[
TV^{\gamma-1} = \text{const}
\]

\[ \Rightarrow T = \frac{T_0 R_0^\delta}{(R_0 + \alpha t)^\delta}, \]

\[ a = C_1 (R_0 + \alpha t) \]

With the assumption of the quasi-static adiabatic expansion and the relation \( a(t) \), we can calculate the condensation for the expansion pion source at each evolving time as static sources.
Condensation fraction

In a canonical ensemble, the total number of bosons

\[ N = N_0 + N_T = \frac{Z}{1 - Z} + \sum_{n>0}^{\infty} \frac{g_n Z e^{-\beta \tilde{E}_n}}{1 - Z e^{-\beta \tilde{E}_n}} \]

\[ \tilde{E}_n = E_n - E_0 \]

\( Z \) — fugacity, which is related to \( E_0 \) and can be solved

\[ f_0 = \frac{N_0}{N} = \frac{Z}{(1 - Z)N} \]

For \( N=2000 \), the system has large condensation fraction, which may reach 0.7 at the temperatures 60-80 MeV.

However, for \( N=1000 \), the system has only small condensation fraction at low temperature for \( C_1 = 0.40 \) than for smaller source \( C_1 = 0.35 \).
In the EPG model, the one- and two-particle density matrices

\[ G^{(1)}(p_1, p_2) = \sum_n u_n^*(p_1)u_n(p_2) \langle \hat{a}_n^+ \hat{a}_n \rangle = \sum_n u_n^*(p_1)u_n(p_2) \frac{g_n Z e^{-\beta E_n}}{1 - Z e^{-\beta E_n}} \]

\[ G^{(2)}(p_1, p_2; p_1, p_2) = \sum_{klmn} u_k^*(p_1)u_l^*(p_2) u_m(p_2)u_n(p_2) \langle \hat{a}_k^+ \hat{a}_l^+ \hat{a}_m \hat{a}_n \rangle \]

The invariant single-pion momentum distribution is

\[ E \frac{dN}{dp} = \sqrt{p^2 + m_{\pi}^2} \ G^{(1)}(p, p) \]

**Two-body density matrix in momentum space**

In the limit of a large number of particles, \( N(N-1) \sim N^2 (\gg N_f, N_0) \)

\[ G^{(2)}(p_1, p_2; p_1, p_2) = G^{(1)}(p_1, p_1)G^{(1)}(p_2, p_2) + |G^{(1)}(p_1, p_2)|^2 - N_0^2 |u_0(p_1)|^2 |u_0(p_2)|^2 \]

**In BE correlation measurement**, we normalize the probability relative to the probability of detecting particle \( p_1 \) and \( p_2 \), and define the momentum correlation function \( C \)

\[ C(p_1, p_2) = \frac{G^{(2)}(p_1, p_2; p_1, p_2)}{G^{(1)}(p_1, p_1)G^{(1)}(p_2, p_2)} \]

\[ C(p, q) = C(p_1, p_2) = 1 + \frac{|G^{(1)}(p_1, p_2)|^2 - N_0^2 |u_0(p_1)|^2 |u_0(p_2)|^2}{G^{(1)}(p_1, p_1)G^{(1)}(p_2, p_2)} \]

This is the general Bose-Einstein correlation for all situations

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Coherent Coherent
Symmetrization absent
No correlation

Coherent Chaotic
Symmetrization
Correlation exist

Chaotic Chaotic
Symmetrization
Correlation exist

Principle of symmetrization
Three-pion correlation functions

\[ C_3(p_1, p_2, p_3) = \frac{G^{(3)}(p_1, p_2, p_3; p_1, p_2, p_3)}{G^{(1)}(p_1; p_1)G^{(1)}(p_2; p_2)G^{(1)}(p_3; p_3)}. \]

\[ C_3(p_1, p_2, p_3) = 1 + R(1, 2) + R(2, 3) + R(1, 3) + R(1, 2, 3) \]

Where

\[ R(i, j) = \frac{|G^{(1)}(p_i, p_j)|^2 - N_0^2|u_0(p_i)|^2|u_0(p_j)|^2}{G^{(1)}(p_i, p_i)G^{(1)}(p_j, p_j)} \]

And

\[ R(i, j, k) = 2\frac{\text{Re}[G^{(1)}(p_i, p_j)G^{(1)}(p_j, p_k)G^{(1)}(p_k, p_i) - N_0^2f_3(p_i, p_j, p_k)\]}{G^{(1)}(p_i, p_i)G^{(1)}(p_j, p_j)G^{(1)}(p_k, p_k)} \]

\[ f_3(p_i, p_j, p_k) = \frac{G^{(1)}(p_i, p_j)u_0(p_i)u_0^*(p_j)}{N_0|u_0(p_i)|^2} + \frac{G^{(1)}(p_j, p_k)u_0(p_j)u_0^*(p_k)}{N_0|u_0(p_j)|^2} + \frac{G^{(1)}(p_k, p_i)u_0(p_k)u_0^*(p_i)}{N_0|u_0(p_k)|^2} - 2\frac{|u_0(p_i)|^2|u_0(p_j)|^2|u_0(p_k)|^2}{N_0}. \]
Three-pion correlation functions

Three-pion correlations as a function of $Q_3$ for the EPG sources with the different temperatures and particle numbers

For the fixed N and T, the correlation functions for the sources with $C_1 = 0.35$ are lower than those for the sources with $C_1 = 0.40$

Because the source with a small $C_1$ has small characteristic length and high condensation fraction

$$Q_3 = \sqrt{Q_{12}^2 + Q_{23}^2 + Q_{13}^2}$$
The widths of the correlation functions in the highest momentum interval are narrower than those in the lowest momentum intervals because the source has a wider spatial distribution for the pions emitted from excited states than that from ground state.
The correlation functions for the sources with $T=80$ MeV are slightly higher than those for the sources with the higher temperatures in the highest momentum interval.

Because the source spatial distribution is narrow at low temperature for the chaotic emission from excited states.
Three-pion cumulant correlation functions

\[ c_3(Q_3) = 1 + R(1, 2, 3) \]

The correlation from the pure pion-triplet interference, \( R(1, 2, 3) \), approaches zero when any pion pair among the three pions is uncorrelated.
Normalized three-pion correlator

Zero-momentum-difference intercept is affected by

- Long-lived resonances
- Particle misidentification
- Experimental binning effect

These effects are cancelled in the "Normalized three-particle correlator"

\[ r_3(123) = \frac{R(1,2,3)}{\sqrt{R(1,2) \times R(2,3) \times R(1,3)}} = 2\cos\Phi \]

For partially coherent sources, the normalized 3-pion correlator at vanishing relative momenta is given by

\[ r_3(p, Q_3 = 0) = \frac{2 \sqrt{\epsilon(p)} [3 - 2\epsilon(p)]}{[2 - \epsilon(p)]^{3/2}} \]

\[ \Phi = \Phi_{12} + \Phi_{23} + \Phi_{31} \]

The intercept of the \( r_3 \) is expected to be 2 in the case of fully chaotic sources

The intercept of the \( r_3 \) is less than 2 in the case of partially coherent sources

\[ \epsilon(p) = \frac{\rho_{ch}(p)}{\rho(p)} \]
**Model results without momentum cut**

For fixed $N$ and $T$, the intercept of three-pion correlators for large sources is higher than that for small sources.
Results with different $k_{T3}$ intervals

Because the uncertainty in isolating the three-pion cumulant when the cumulant correlation itself is small, $c_3 \sim 1.0$

$r_3(Q_3)$ is largely uncertain for the more central collisions
Classifications of Four-pion correlations

- **Partial cumulant**
  - Double pairs, Triplets, Quadruplets
    - $a_4(Q_4)$

- **Cumulant Correlation**
  - Triplets, Quadruplets
    - $b_4(Q_4)$
  - Only Quadruplets
    - $c_4(Q_4)$

- **Full correlation**
  - Single, double pairs, Triplets, Quadruplets
    - $c_4(Q_4)$
Mathematical expressions

1:

\[ C_4(p_1, p_2, p_3, p_4) = 1 + R(1, 2) + R(1, 3) + R(1, 4) + R(2, 3) + R(2, 4) + R(3, 4) + R(1, 2)R(3, 4) + R(1, 3)R(2, 4) + R(1, 4)R(2, 3) + R(1, 2, 3) + R(1, 2, 4) + R(1, 3, 4) + R(2, 3, 4) + R(1, 2, 3, 4) + R(1, 2, 4, 3) + R(1, 3, 2, 4) \]

2:

\[ a_4(p_1, p_2, p_3, p_4) = 1 + R(1, 2)R(3, 4) + R(1, 3)R(2, 4) + R(1, 4)R(2, 3) + R(1, 2, 4) + R(1, 3, 4) + R(1, 2, 3) + R(2, 3, 4) + R(1, 2, 3, 4) + R(1, 2, 4, 3) + R(1, 3, 2, 4) \]

3:

\[ b_4(p_1, p_2, p_3, p_4) = 1 + R(1, 2, 3) + R(1, 2, 4) + R(1, 3, 4) + R(2, 3, 4) + R(1, 2, 3, 4) + R(1, 2, 4, 3) + R(1, 3, 2, 4) \]

4:

\[ c_4(p_1, p_2, p_3, p_4) = 1 + R(1, 2, 3, 4) + R(1, 2, 4, 3) + R(1, 3, 2, 4) \]
Higher order QS correlations contain additional information about the source which cannot be learned from 2- and 3-particle correlations alone.

Four-pion correlations are higher than three-pion correlation functions because there are more contributions of the correlations of single pion pair, double pion pair, pure pion triplet and quadruplet.

\[ Q_4 = \sqrt{q_{12}^2 + q_{13}^2 + q_{14}^2 + q_{23}^2 + q_{24}^2 + q_{34}^2} \]
Four-pion correlations with different $k_{T4}$ intervals

Follow experimental momentum cuts

Large number of particles and low temperature welcome to the coherence in case of small source

This finite condensation may decrease the correlation functions at low pion transverse momenta, but influence only slightly at high pion transverse momentum

\[ K_{T4} = \frac{|p_{T1} + p_{T2} + p_{T3} + p_{T4}|}{4} \]
Partial cumulant correlation functions

Correlations of single pair are removed

Correlations of single and double pair are removed
Cumulant correlation functions

\[ c_4(p_1, p_2, p_3, p_4) = 1 + R(1, 2, 3, 4) + R(1, 2, 4, 3) + R(1, 3, 2, 4) \]

- \( c_4(Q_4) \) is more sensitive to source condensation as compared to other partials correlations.
Normalized four-pion correlator

The 4-pion FT phase factor may be isolated by comparing 4-pion cumulant to 2-pion QS correlations

\[ r_4(1, 2, 3, 4) = \frac{c_4(1, 2, 3, 4) - 1}{\sqrt{C_2(1, 2) \times C_2(2, 3) \times C_2(3, 4) \times C_2(1, 4)}} = 6 \cos \Phi \]

For partially coherent sources, the normalized 4-pion correlator at vanishing relative momenta is given by

\[ r_4(p, Q_4 = 0) = \frac{6 \ \epsilon(p) [4 - 3 \epsilon(p)]}{[2 - \epsilon(p)]^2} \]

\[ \epsilon(p) = \frac{\rho_{ch}(p)}{\rho(p)} \]

The intercept of the \( r_4 \) is expected to be 6 in the case of fully chaotic sources

The intercept of the \( r_4 \) is less than 6 in the case of partially coherent sources
The suppression of four-pion as compared to two-pion measured by $r_4$ seems to suggest a finite coherent component to the pion production in heavy-ion collisions.
Results with different $k_{T4}$ intervals

No significant $Q_4$ dependence was observed for both low and high transverse momentum
Model results with experimental data
Model results and experimental data

This may be because the average longitudinal momentum of the three pions in the spherical EPG model is smaller than that in the experiment in low transverse-momentum intervals.
In the small $Q_4$ region the experimental results are between the model results for the sources with the small and large $N$.

This may be because the average longitudinal momentum of the four pions in the spherical EPG model is smaller than that in the experiment in low transverse-momentum intervals.
The model results are independent of the particle number of the source and consistent with the experimental data in high transverse momentum.
Extracted condensation fractions

The values of condensation fraction are determined by the comparisons of the model results and experimental data of three- and four–pion correlations.

They are consistent with the value of coherent fraction extracted by the ALICE collaboration.

In the EPG model the source size for small $C_1$ parameter is smaller than that for a larger $C_1$ parameter.

Considering the source size is larger for central collisions than that for peripheral collisions in the experiments.
Summary and Conclusion

We have calculated three and four-pion correlation functions and found that they are sensitive to the condensation fraction of EPG sources at low transverse momentum.

Our model calculation results may explain the experimental data of both the three- and four-pion correlation functions.

According to the EPG model, the condensation not only depends on the particle number but also depends on the source size which is also smaller in peripheral collisions than in the central collisions.

So the comprehensive effect of particle number and source size may lead to the result that the condensation fraction is independent of collision centrality.

The source condensation fraction determine by the comparison is between 16% and 47%.

Normalized three- and four-pion correlators are more sensitive to the condensation fractions as compared to the simple correlation functions.
Thanks for your attention!
The RMSR of the sources for N=2000. Considering that the RMSR in Pb-Pb collisions at LHC to be on the order 10 fm, the value of parameter C1 being between 0.35 and 0.40 appear reasonable.

\[ I = 2\sqrt{1 - G \frac{1 + 2G}{(1 + G)^{3/2}}} \]